Chapter 8

Influence Lines for Beams (Contd.)

Instructional Objectives:

The objectives of the present lesson are as follows.

- Construction of influence line for maximum shear at sections in a beam supporting two concentrated loads
- Construction of influence line for maximum moment at sections in a beam supporting two concentrated loads
- Construction of influence line for maximum end shear in a beam supporting a series of moving concentrated loads
- Construction of influence line for maximum shear at a section in a beam supporting a series of moving concentrated loads
- Construction of influence line for maximum moment at a section in a beam supporting a series of moving concentrated loads
- Construction of influence line for absolute maximum moment in s beam supporting a series of moving concentrated loads
- Understanding about the envelopes of maximum influence line values

Introduction

In the previous lessons, we have studied about construction of influence line for either single concentrated load or uniformly distributed loads. In the present lesson, we will study in depth about the beams, which are loaded with a series of two or more than two concentrated loads.

Maximum shear at sections in a beam supporting two concentrated loads

Let us assume that instead of one single point load, there are two point loads P_1 and P_2 spaced by a distance y moving from left to right on the beam as shown in Figure 8.1. We are interested to find maximum shear force in the beam at given section C. In the present case, we assume that $P_2<P_1$.

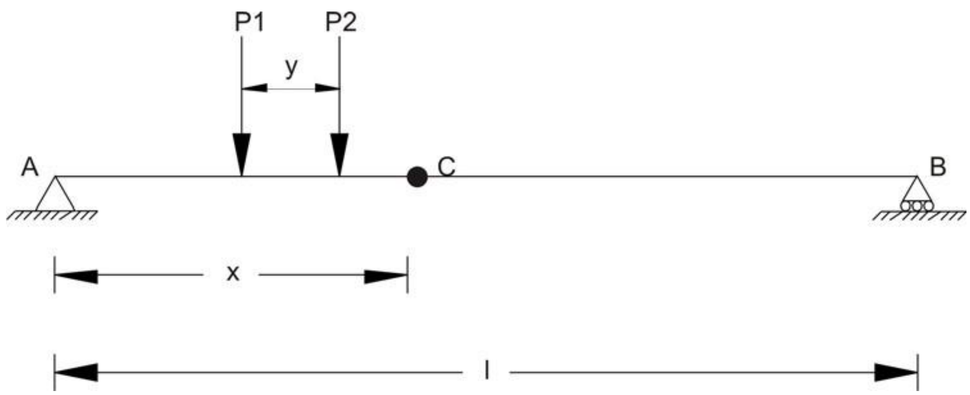

Figure 8.1: Beam loaded with two concentrated point loads

Now there are three possibilities due to load spacing.
They are: x<y, x=y and x>y.

Case 1: x<y

This case indicates that when load P_2 will be between A and C then load P_1 will not be on the beam. In that case, maximum negative shear at section C can be given by

$$V_C = -P_2 \frac{x}{l}$$

and maximum positive shear at section C will be

$$V_C = P_2 \frac{(l-x)}{l}$$

Case 2: x=y

In this case, load P_1 will be on support A and P_2 will be on section C. Maximum negative shear can be given by

$$V_C = -P_2 \frac{x}{l}$$

and maximum positive shear at section C will be

$$V_C = P_2 \frac{(l-x)}{l}$$

Case 3: x>y

With reference to Figure 8.2, maximum negative shear force can be obtained when load P_2 will be on section C. The maximum negative shear force is expressed as:

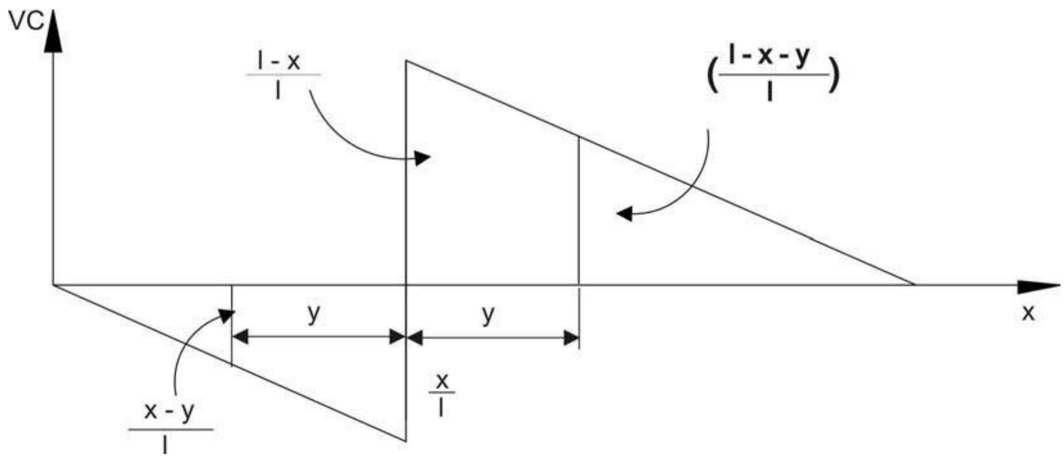

Figure 8.2: Influence line for shear at section C

$$V_C^{\,1} = -P_2 \frac{x}{l} - P_1\left(\frac{x-y}{l}\right)$$

And with reference to Figure 8.2, maximum positive shear force can be obtained when load P_1 will be on section C. The maximum positive shear force is expressed as:

$$V_C^{\,2} = -P_1 \frac{x}{l} + P_2\left(\frac{l-x-y}{l}\right)$$

From above discussed two values of shear force at section, select the maximum negative shear value.

Maximum moment at sections in a beam supporting two concentrated loads

Let us assume that instead of one single point load, there are two point loads P_1 and P_2 spaced at y moving left to right on the beam as shown in Figure 8.3. We are interested to find maximum moment in the beam at given section C.

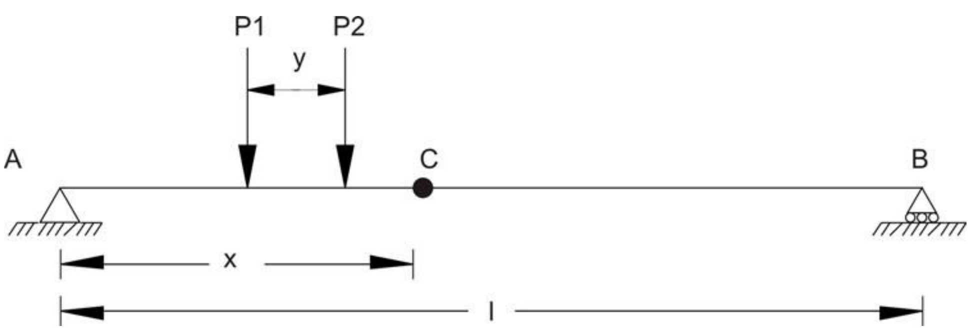

Figure 8.3: Beam loaded with two concentrated loads

With reference to Figure 8.4, moment can be obtained when load P_2 will be on section C. The moment for this case is expressed as:

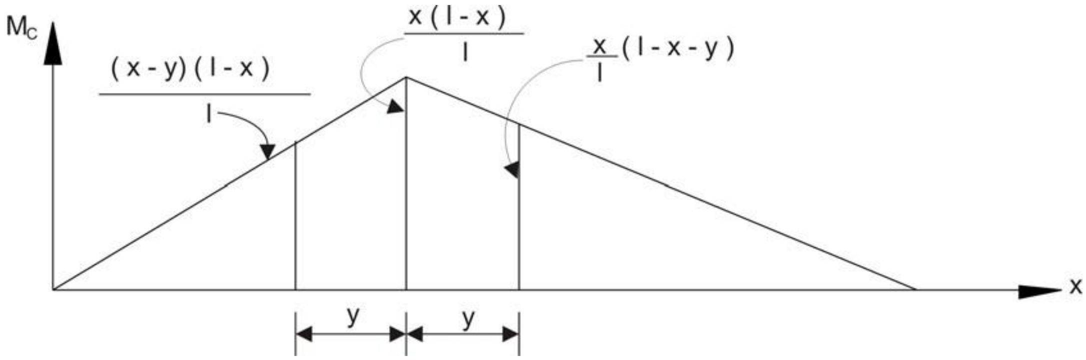

Figure 8.4: Influence line for moment at section C

$$M_C{}^1 = P_1(x-y)\left(\frac{l-x}{l}\right) + P_2 x\left(\frac{l-x}{l}\right)$$

With reference to Figure 8.4, moment can be obtained when load P_1 will be on section C. The moment for this case is expressed as:

$$M_C{}^2 = P_1 x\left(\frac{l-x}{l}\right) + P_2 x\left(\frac{l-x-y}{l}\right)$$

From above two cases, maximum value of moment should be considered for maximum moment at section C when two point loads are moving from left end to right end of the beam.

Maximum end shear in a beam supporting a series of moving concentrated loads

In real life situation, usually there are more than two point loads, which will be moving on bridges. Hence, in this case, our aim is to learn, how to find end shear in beam supporting a series of moving concentrated loads. Let us assume that as shown in Figure 8.5, four concentrated loads are moving from right end to left end on beam AB. The spacing of the concentrated load is given in Figure 8.5.

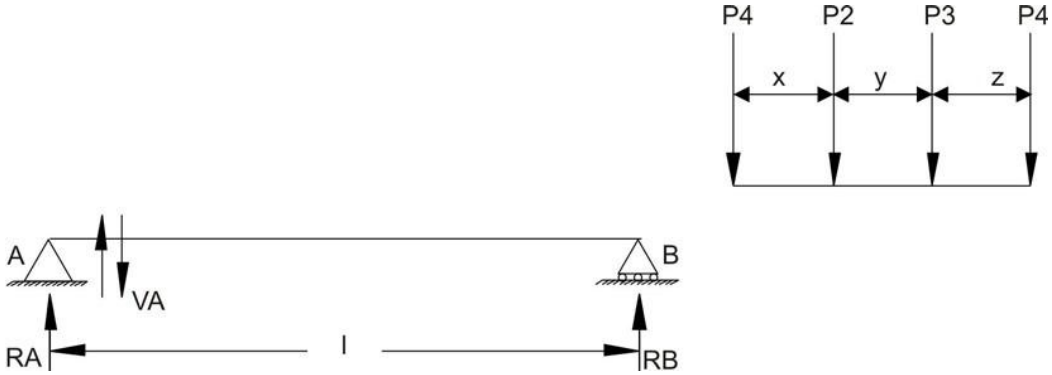

Figure 8.5: Beam loaded with a series of loads

As shown in figure, we are interested in end shear at A. We need to draw influence line for the support reaction A and a point away from the support at infinitesimal distance on the span for the shear V_A. The influence lines for these cases are shown in Figure 8.6 and 8.7.

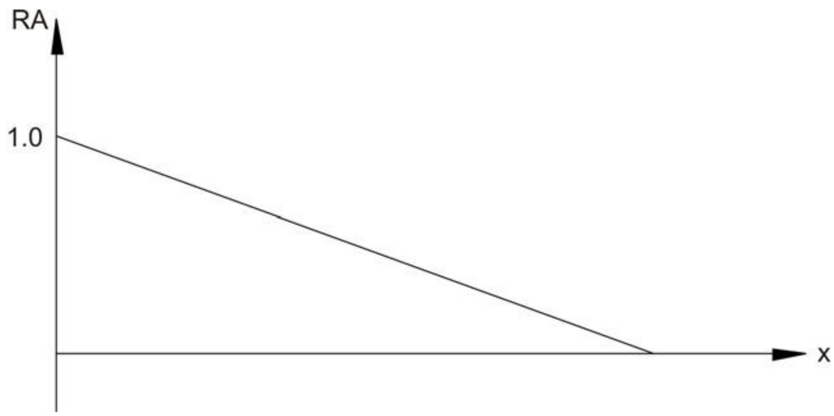

Figure 8.6: Influence line for reaction at support A

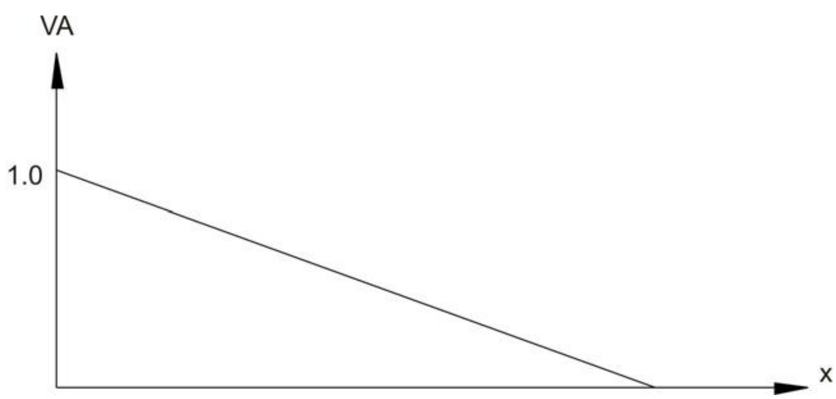

Figure 8.7: Influence line for shear near to support A.

When loads are moving from B to A then as they move closer to A, the shear value will increase. When load passes the support, there could be increase or decrease in shear value depending upon the next point load approaching support A. Using this simple logical approach, we will find out the change in shear value near support and monitor this change from positive value to negative value. Here for the present case let us assume that ΣP is summation of the loads remaining on the beam. When load P_1 crosses support A, then P_2 will approach A. In that case, change in shear will be expressed as

$$dV = \frac{\sum Px}{l} - P_1$$

When load P_2 crosses support A, then P_3 will approach A. In that case change in shear will be expressed as

$$dV = \frac{\sum Py}{l} - P_2$$

In case if *dV* is positive then shear at A has increased and if *dV* is negative, then shear at A has decreased. Therefore, first load, which crosses and induces negative changes in shear, should be placed on support A.

Numerical Example

Compute maximum end shear for the given beam loaded with moving loads as shown in Figure 8.8.

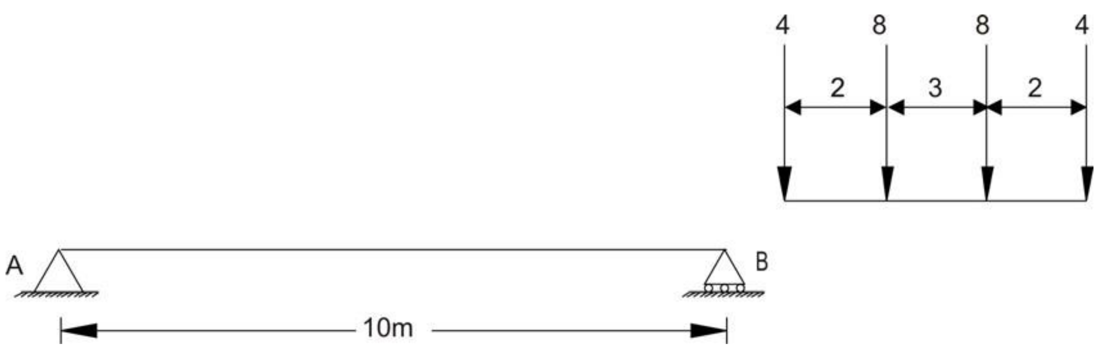

Figure 8.8: Beam loaded with a series of four concentrated loads

When first load of 4 kN crosses support A and second load 8 kN is approaching support A, then change in shear can be given by

$$dV = \frac{\sum(8+8+4)2}{10} - 4 = 0$$

When second load of 8 kN crosses support A and third load 8 kN is approaching support A, then change in shear can be given by

$$dV = \frac{\sum(8+4)3}{10} - 8 = -3.8$$

Hence, as discussed earlier, the second load 8 kN has to be placed on support A to find out maximum end shear (refer Figure 8.9).

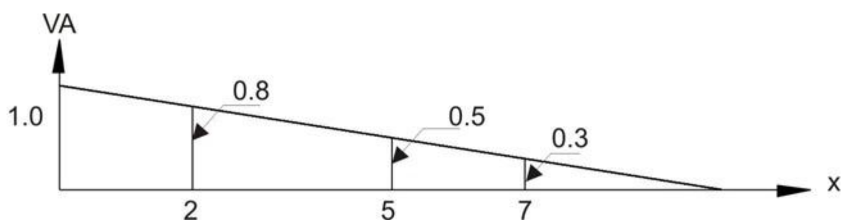

Figure 8.9: Influence line for shear at A.

$$V_A = 4 \times 1 + 8 \times 0.8 + 8 \times 0.5 + 4 \times 0.3 = 15.6kN$$

Maximum shear at a section in a beam supporting a series of moving concentrated loads

In this section we will discuss about the case, when a series of concentrated loads are moving on beam and we are interested to find maximum shear at a section. Let us assume that series of loads are moving from right end to left end as shown in Figure. 8.10.

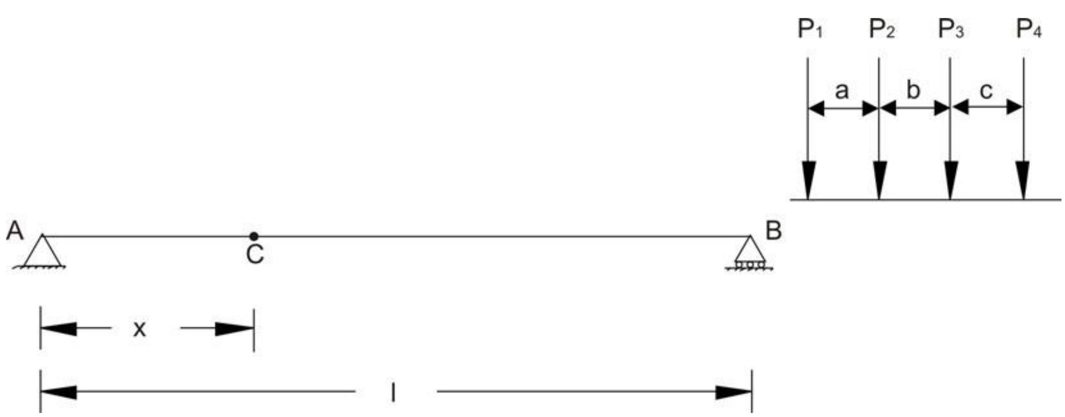

Figure 8.10: Beam loaded with a series of loads

The influence line for shear at the section is shown in Figure 8.11.

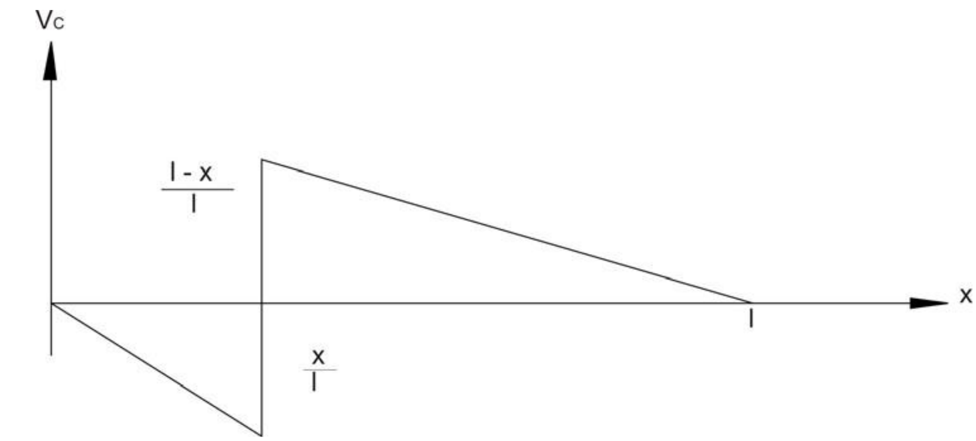

Figure 8.11: Influence line for shear at section C

Monitor the sign of change in shear at the section from positive to negative and apply the concept discussed in earlier section. Following numerical example explains the same.

Numerical Example

The beam is loaded with concentrated loads, which are moving from right to left as shown in Figure 8.12. Compute the maximum shear at the section C.

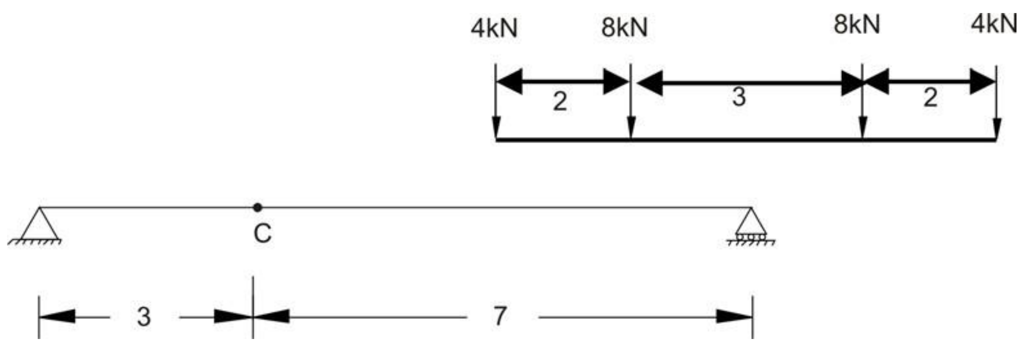

Figure 8.12: Beam loaded with a series of loads

The influence line at section C is shown in following Figure 8.13.

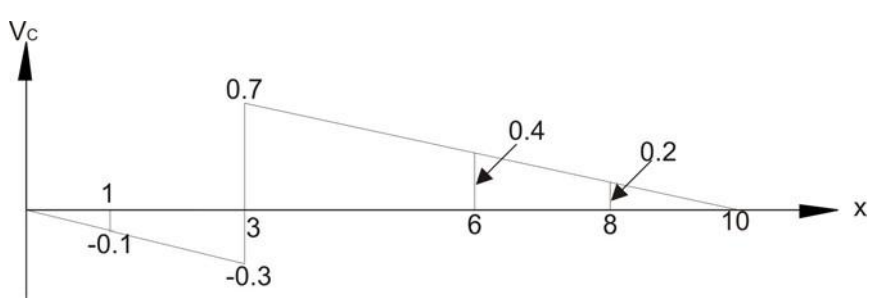

Figure 8.13: Influence line for shear at section C

When first load 4kN crosses section C and second load approaches section C then change in shear at a section can be given by

$$dV = \frac{20 \times 2}{10} - 4 = 0$$

When second load 8 kN crosses section C and third load approaches section C then change in shear at section can be given by

$$dV = \frac{12 \times 3}{10} - 8 = -4.4$$

Hence place the second concentrated load at the section and computed shear at a section is given by

$$V_C == 0.1\times 4 + 0.7 \times 8 + 0.4 \times 8 + 0.2 \times 4 = 9.2kN$$

Maximum Moment at a section in a beam supporting a series of moving concentrated loads

The approach that we discussed earlier can be applied in the present context also to determine the maximum positive moment for the beam supporting a series of moving concentrated loads. The change in moment for a load P_1 that moves from position x_1 to x_2 over a beam can be obtained by multiplying P_1 by the change in ordinate of the influence line i.e. $(y_2 - y_1)$. Let us assume the slope of the influence line (Figure 8.14) is S, then $(y_2 - y_1) = S\,(x_2 - x_1)$.

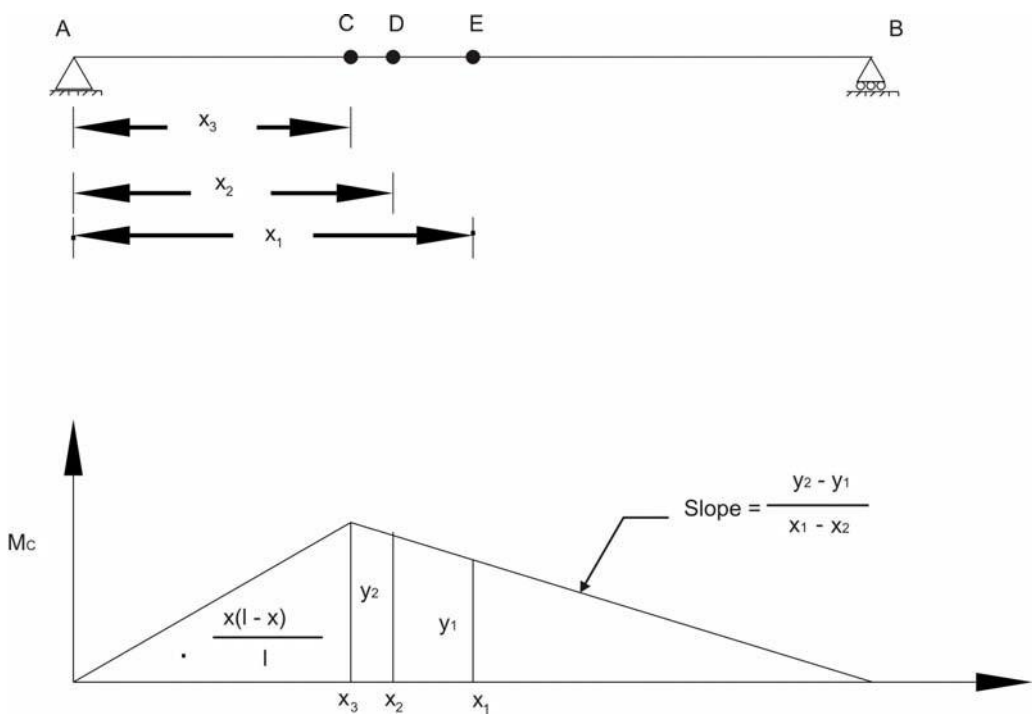

Figure 8.14: Beam and Influence line for moment at section C

Hence change in moment can be given by

$$dM = P_1 S(x_2 - x_1)$$

Let us consider the numerical example for better understanding of the developed concept.

Numerical Example

The beam is loaded with concentrated loads, which are moving from right to left as shown in Figure 8.15. Compute the maximum moment at the section C.

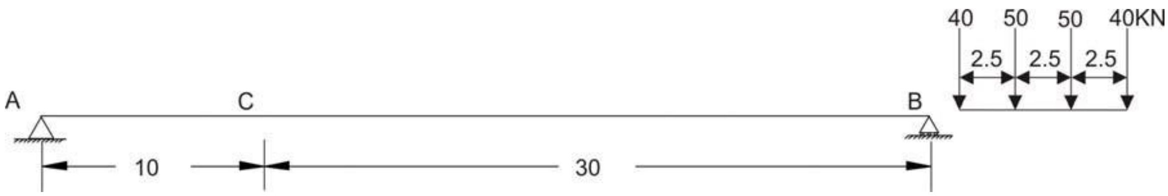

Figure 8.15: Beam loaded with a series of loads

The influence line for moment at C is shown in Figure 8.16.

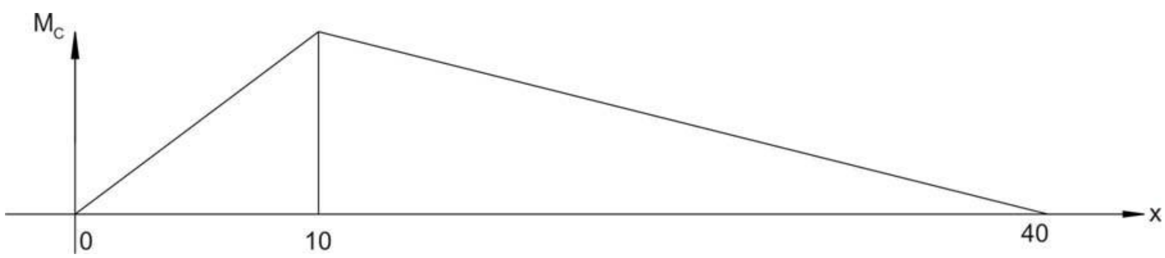

Figure 8.16: Beam loaded with a series of loads

If we place each of the four-concentrated loads at the peak of influence line, then we can get the largest influence from each force. All the four cases are shown in Figures 8.17-20.

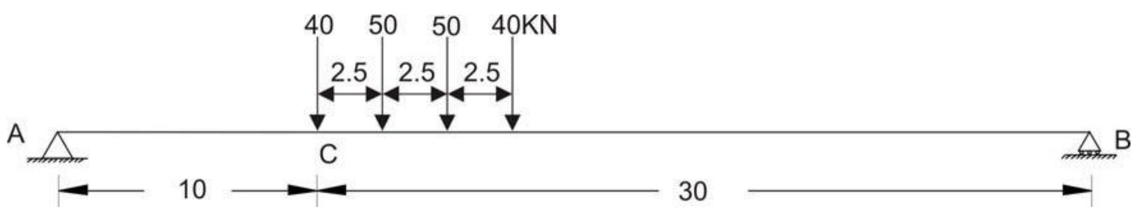

Figure 8.17: Beam loaded with a series of loads – First load at section C

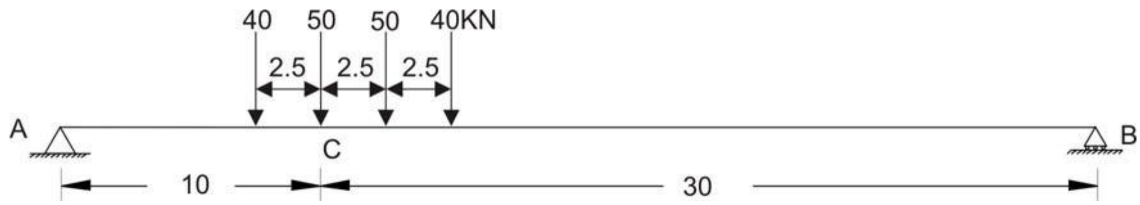

Figure 8.18: Beam loaded with a series of loads – Second load at section C

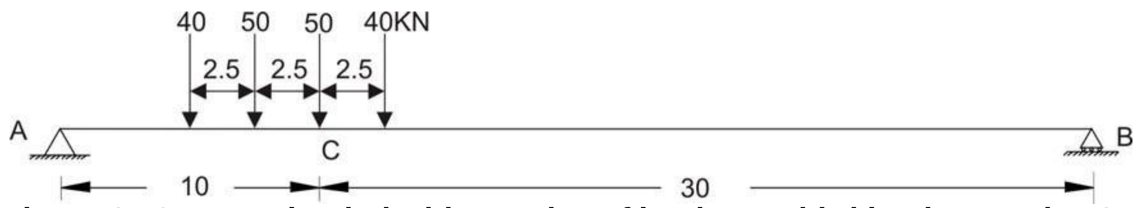

Figure 8.19: Beam loaded with a series of loads - – Third load at section C

Figure 8.20: Beam loaded with a series of loads - – Fourth load at section C

As shown in Figure 8.17, when the first load crosses the section C, it is observed that the slope is downward (7.5/10). For other loads, the slope is upward (7.5/(40-10)). When the first load 40 kN crosses the section and second load 50 kN is approaching section (Figure 8.17) then change in moment is given by

$$dM = -40\left(\frac{7.5}{10}\right)2.5 + (50+50+40)\left(\frac{7.5}{(40-10)}\right)2.5 = 12.5kN.m$$

When the second load 50 kN crosses the section and third load 50 kN is approaching section (Figure 8.18) then change in moment is given by

$$dM = -(40+50)\left(\frac{7.5}{10}\right)2.5 + (50+40)\left(\frac{7.5}{(40-10)}\right)2.5 = -112.5kN.m$$

At this stage, we find negative change in moment; hence place second load at the section and maximum moment (refer Figure 8.21) will be given by

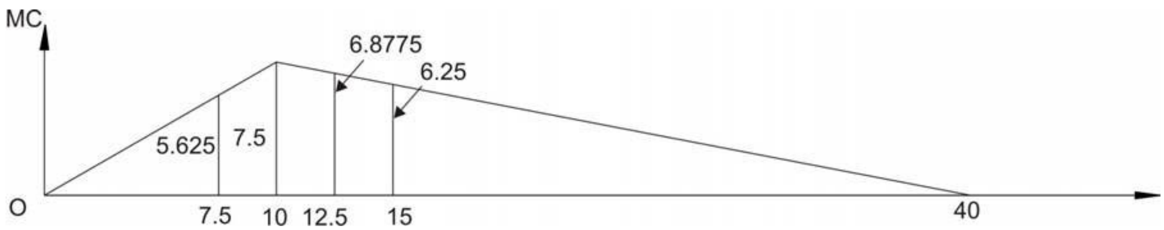

Figure 8.21: Influence line for moment at C

$$M_c = 40(5.625) + 50(7.5) + 50(6.8775) + 40(6.25) = 1193.87 kNm$$

Absolute maximum moment in s beam supporting a series of moving concentrated loads.

In earlier sections, we have learned to compute the maximum shear and moment for single load, UDL and series of concentrated loads at specified locations. However, from design point of view it is necessary to know the critical location of the point in the beam and the position of the loading on the beam to find maximum shear and moment induced by the loads. Following paragraph explains briefly for the cantilever beam or simply supported beam so that quickly maximum shear and moment can be obtained.

Maximum Shear: As shown in the Figure 8.22, for the cantilever beam, absolute maximum shear will occur at a point located very near to fixed end of the beam. After placing the load as close as to fixed support, find the shear at the section close to fixed end.

Figure 8.22: Absolute maximum shear case – cantilever beam

Similarly for the simply supported beam, as shown in Figure 8.23, the absolute maximum shear will occur when one of the loads is placed very close to support.

Figure 8.23: Absolute maximum shear – simply supported beam

Moment:

The absolute maximum bending moment in case of cantilever beam will occur where the maximum shear has occurred, but the loading position will be at the free end as shown in Figure 8.24.

Figure 8.24: Absolute maximum moment – cantilever beam

The absolute maximum bending moment in the case of simply supported beam, one cannot obtain by direct inspection. However, we can identify position analytically. In this regard, we need to prove an important proposition.

Proposition:

When a series of wheel loads crosses a beam, simply supported ends, the maximum bending moment under any given wheel occurs when its axis and the center of gravity of the load system on span are equidistant from the center of the span.

Let us assume that load P_1, $P_{2,}$ P_3 etc. are spaced shown in Figure 8.25 and traveling from left to right. Assume P_R to be resultant of the loads, which are on the beam, located in such way that it nearer to P_3 at a distance of d_1 as shown in Figure 8.25.

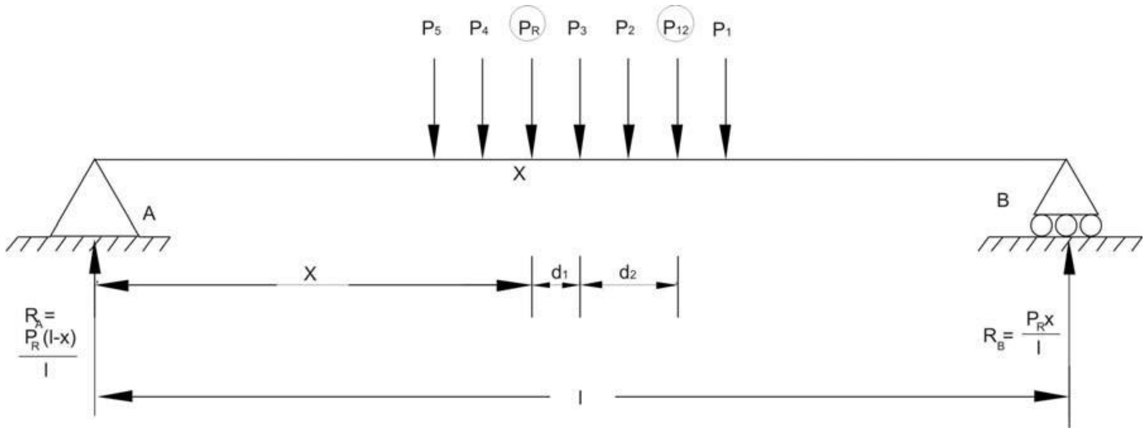

Figure 8.25: Absolute maximum moment case – simply supported beam

If P_{12} is resultant of P_1 and P_2, and distance from P_3 is d_2. Our objective is to find the maximum bending moment under load P_3. The bending moment under P_3 is expressed as

$$M = \frac{P_R x}{l}(l - x - d_1) - P_{12}(d_2)$$

Differentiate the above expression with respect to x for finding out maximum moment.

$$\frac{dM}{dx} = \frac{P_R}{l}(l - 2x - d_1) = 0 \Rightarrow l - 2x + d_1 = 0 \Rightarrow x = \frac{l}{2} - \frac{d_1}{2}$$

Above expression proves the proposition.

Let us have a look to some examples for better understanding of the above-derived proposition.

Numerical Examples

Example 1:

The beam is loaded with two loads 25 kN each spaced at 2.5 m is traveling on the beam having span of 10 m. Find the absolute maximum moment

Solution:

When the a load of 25kN and center of gravity of loads are equidistant from the center of span then absolute bending moment will occur. Hence, place the load on the beam as shown in Figure 8.26.

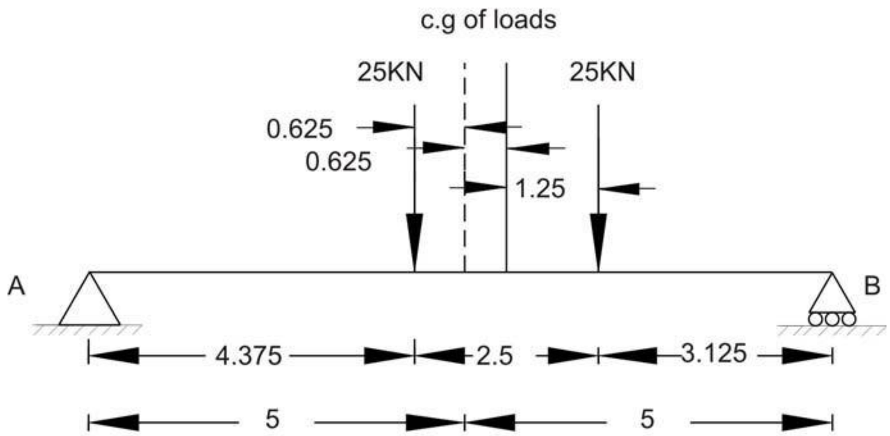

Figure 8.26: Simply supported beam (Example 1)

The influence line for M_x is shown in Figure 8.27

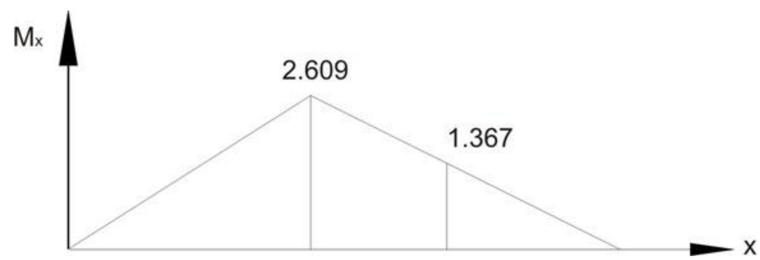

Figure 8.27: Influence line for moment at X (Example 1)

Computation of absolute maximum moment is given below.

$$M_x = 25(2.461) + 25(1.367) = 95.70 kN.m$$

Example 2:

Compute the absolute maximum bending moment for the beam having span of 30 m and loaded with a series of concentrated loads moving across the span as shown in Figure 8.28.

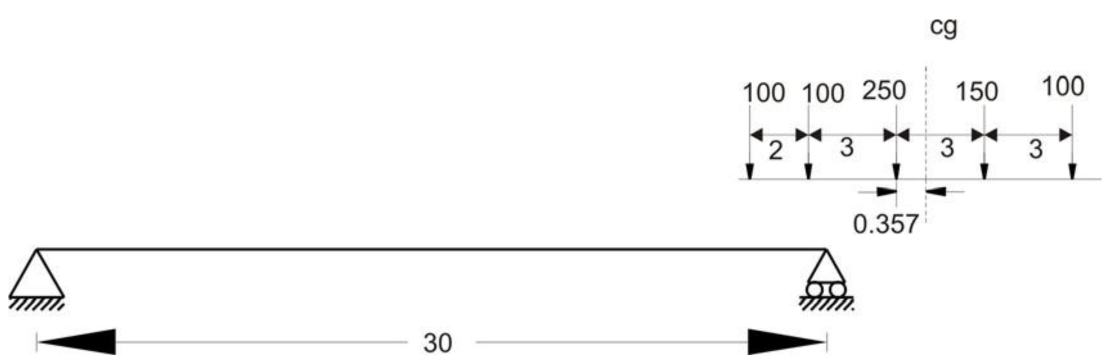

Figure 8.28: Simply supported beam (Example 2)

First of all compute the center of gravity of loads from first point load of 100 kN

$$= \frac{100(2)+250(5)+150(8)+100(11)}{100+100+250+150+100} = \frac{3750}{700} = 5.357m$$

$=$

Now place the loads as shown in Figure 8.29.

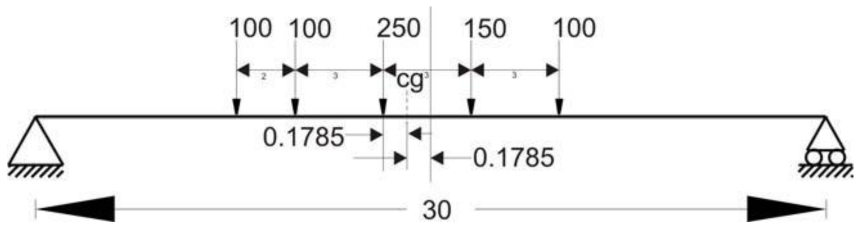

Figure 8.29: Simply supported beam with load positions (Example 2)

Also, draw the influence line as shown in Figure 8.30 for the section X.

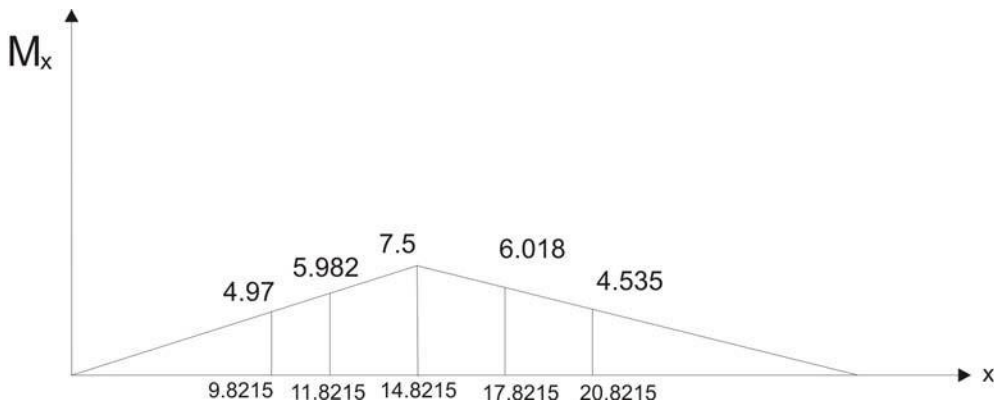

Figure 8.30: Influence Line for moment at section X (Example 2)

$$M_x = 100(4.97) + 100(5.982) + 250(7.5) + 150(6.018) + 100(4.535) = 4326.4kN.m$$

Envelopes of maximum influence line values

For easy calculations steps of absolute maximum shear and moment rules for cantilever beam and simply supported beam were discussed in previous section. Nevertheless, it is difficult to formulate such rules for other situations. In such situations, the simple approach is that develop the influence lines for shear and moment at different points along the entire length of the beam. The values easily can be obtained using the concepts developed in earlier sections. After obtaining the values, plot the influence lines for each point under consideration in one plot and the outcome will be envelop of maximums. From this diagram, both the absolute maximum value of shear and moment and location can be obtained. However, the approach is simple but demands tedious calculations for each point. In that case, these calculations easily can be done using computers.

Closing Remarks

In this lesson, we have learned various aspects of constructing influence lines for the cases when the moving concentrated loads are two or more than two. Also, we developed simple concept of finding out absolute maximum shear and moment values in cases of cantilever beam and simply supported beam. Finally, we discussed about the need of envelopes of maximum influence line values for design purpose.

Suggested Text Books for Further Reading

- Hibbeler, R. C. *Structural Analysis*, Pearson Education (Singapore) Pte. Ltd., Delhi, ISBN 81-7808-750-2

- Junarkar, S. B. and Shah, H. J. *Mechanics of Structures – Vol. II*, Charotar Publishing House, Anand.